KPI 9.01 Decimals Manipulation

1) Multiplying decimals	1) Remove the decimal points. 2) Multiply. 3) Insert the same number of decimal points in the answer as in the question. 0.5 x 0.3 5 x 3 = 15 0.5 x 0.3 = 0.15	**2) Dividing a decimal by an integer**	0.72÷6 $6\overline{)0.72}$ = 0.12 0.972÷8 $8\overline{)0.9720}$ = 0.1215
		3) Dividing an integer by a decimal	1) Write as a fraction. 2) Form an equivalent fraction. 3) Divide.

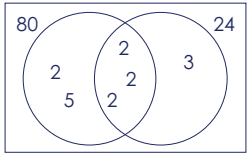
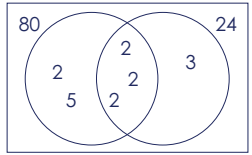
KPI 9.02 Estimation and Limits of Accuracy

1) ≈	"approximately equal to"	**2) Truncation**	Ignoring all decimal places past a certain point without rounding.
3) Significant figures	The total number of digits in a number, not counting the zeros at the beginning of a number or at the end of a decimal number. 345 000 has 6 significant figures. 0.3047 has 4 significant figures.	**4) Estimate**	Find approximate answer by calculating with numbers rounded to one significant figure.
5) Error Intervals	The range of values (between the upper and lower bounds) in which the precise value could be. **least possible value ≤ x < greatest possible value**		

KPI 9.03 Related Calculations

19 x 18 = 342	108 ÷ 9 = 12
19 x 180 = 3420	1080 ÷ 9 = 120
190 x 18 = 3420	108 ÷ 90 = 1.2
190 x 180 = 34200	108 ÷ 0.9 = 120
1900 x 180 = 342000	108 ÷ 0.09 = 1200

KPI 9.04 HCF and LCM of Large Numbers

1) Prime numbers	A prime number has two distinct factors; 1 and itself. 2 is the only even prime number. 1 is not a prime number. The first ten prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29		
2) Factor	Any whole number that divides exactly into another number leaving no remainder. Factors of 20 are: 1, 2, 4, 5, 10, 20	**3) Multiple**	The result of multiplying a number with a whole number. (times tables!) The multiples of 7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70 ...
4) HCF–Venn diagram	 HCF of 80 and 24 = 2 x 2 x 2 = 8	**5) LCM–Venn diagram**	 LCM of 80 and 24 = 2 x 2 x 2 x 2 x 3 x 5 = 240

KPI 9.05 Fraction Calculations

1) Writing one number as a fraction of another	Write £15 as a fraction of £25. $\frac{15}{25} = \frac{3}{5}$	**2) Reciprocal**	*Reciprocal of* $7 \rightarrow \frac{1}{7}$ *Reciprocal of* $\frac{2}{3} \rightarrow \frac{3}{2}$
3) Fractions of an amount	Divide the amount by the denominator and then multiply the result by the numerator.		
4) Add/Subtract fractions	Make the denominators the same (find the LCM). Use equivalent fractions to change each fraction to the common denominator. Add/subtract the numerators only.		$\frac{2}{7} + \frac{2}{5} = \frac{10}{35} + \frac{14}{35} = \frac{24}{35}$
5) Multiplying fractions	Multiply the numerators. Multiply the denominators. Simplify where possible.		$\frac{4}{5} \times \frac{3}{8} = \frac{12}{40} = \frac{3}{10}$
6) Dividing fractions	Keep the first fraction the same. Change the second to its reciprocal. Multiply the fractions. Simplify/convert to mixed number where possible.		$\frac{4}{5} \div \frac{3}{8} = \frac{4}{5} \times \frac{8}{3} = \frac{32}{15} = 2\frac{2}{15}$

KPI 9.06 Algebraic Manipulation

1) 2a	2 x a	**2) ab**	a x b
3) a^2	a x a	**4) $3a^2$**	3 x a x a
5) a subtracted from b	b - a	**6) a less than b**	b - a
7) a divided by b	$\frac{a}{b}$	**8) b divided by a**	$\frac{b}{a}$
9) 4 times smaller than a	$\frac{a}{4}$ or $a \div 4$	**10) 4 times larger than a**	4 x a → 4a
11) 5th power of a	a^5	**12) Variable**	A letter used to represent any number.
13) Coefficient	The number to the left of the variable. This is the value that we multiply the variable by. $4x$ → The coefficient of x is 4. x → The coefficient of x is 1.	**14) Term**	A single number, variable or numbers and variables multiplied together.
15) Simplifying	An expression can be simplified by grouping like terms. E.g. $2a + b^2 - 4b + 7a = 9a+b^2 - 4b$	**16) Identity**	An identity is an equation which is always true no matter what value of the unknown is substituted. E.g. $3x - 15 = 3(x - 5)$

KPI 9.07 Index Laws

1) Multiplication law	$a^m \times a^n = a^{m+n}$ Same base numbers, ADD the powers.	**2) Division law**	$a^m \div a^n = a^{m-n}$ Same base numbers, SUBTRACT the powers.
3) Power to a power	$(a^m)^n = a^{m \times n}$ MULTIPLY the powers.	**4) Raising a fraction by a power**	$(ab)^n = a^n \times b^n$ Raise each number or variable to the same power.
5) Power of 0	$a^0 = 1.$ Any number or variable to the power of zero equals 1.	**6) Negative powers (integers)**	$a^{-1} = \frac{1}{a}$ $a^{-2} = \frac{1}{a^2}$ $a^{-n} = \frac{1}{a^n}$ A negative power represents the reciprocal.
7) Positive unit fractions	$a^{\frac{1}{2}} = \sqrt{a}$ $a^{\frac{1}{3}} = \sqrt[3]{a}$ $a^{\frac{1}{n}} = \sqrt[n]{a}$	**8) Negative unit fractions**	$a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}$ $a^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{a}}$ $a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$
9) Positive non-unit fractions	$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$	**10) Negative non-unit factions**	$(a)^{-\frac{m}{n}} = \left(\frac{1}{a}\right)^{\frac{m}{n}} = \left(\sqrt[n]{\frac{1}{a}}\right)^m$

9.08 Standard Form

1) Rule	Numbers written in standard form are always written in the form $a \times 10^n$, where $0<a<10$	**2) Powers of 10**	$10^1 = 10$ $10^2 = 100$ $10^3 = 1000$ $10^3 = 10\,000$ $10^4 = 100\,000$ etc.	$10^{-1} = \frac{1}{10} = 0.1$ $10^{-2} = \frac{1}{100} = 0.01$ $10^{-3} = \frac{1}{1000} = 0.001$ $10^{-4} = \frac{1}{10000} = 0.0001$ etc
3) Ordinary to Standard Form	$340000 = 3.4 \times 10^5$ $0.00903 = 9.03 \times 10^{-3}$	**4) Standard Form to Ordinary**	$1.09 \times 10^3 = 1090$ $8.77 \times 10^{-6} = 0.00000877$	

KPI 9.09 Expanding and Factorising 2

1) Expand	Multiply out the bracket(s) in the expression. E.g. $3(5x + 7) = 15x + 21$	**2) Factorise**	Identify the HCF and rewrite the expression with brackets. E.g. $6x^2 + 9x = 3x(2x+3)$.
3) Expanding double brackets	Writing two brackets next to each other means the brackets need to be multiplied together. $(x + 1)(x + 2) = (x + 1) \times (x + 2) = x^2 + 3x + 2$ Note: $(x + a)^2 = (x + a)(x + a)$		
4) Factorising quadratics	To factorise a quadratic, put it back into a pair of brackets. To find the terms that go in each bracket, look for a pair of numbers which multiply to give the constant and add together to give the coefficient of x		
5) Difference of two squares (DOTS)	$a^2 - b^2 = (a+b)(a-b)$	E.g. $x^2 - 16 = (x + 4)(x - 4)$	

x	x	+1
x	x^2	$+x$
+2	$+2x$	+2

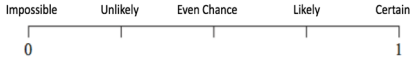
KPI 9.10 Forming Expressions and Substitution

1) Substitution	Replace a variable with a given value.	**2) Function machine**	Shows the relationship between two variables, the input and the output.
3) Formula	A mathematical relationship or rule expressed in symbols.		
4) Expression	A mathematical statement which contains one or more terms combined with addition and/or subtraction signs.		

KPI 9.11 Direct and inverse proportion

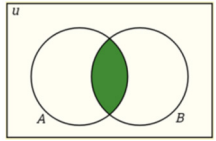
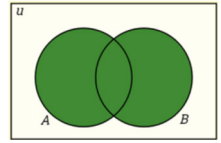
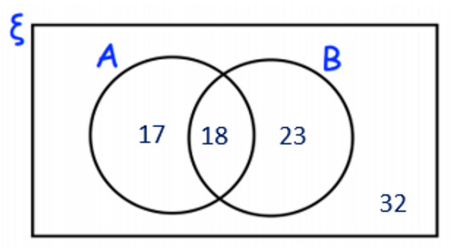
1) Direct proportion	A relationship between two variables where, as one increases, the other also increases.	**3) Unitary method**	To find the value of one unit first.
2) Inverse Proportion	A relationship between two variables where, as one increases, the other decreases.	**4) Exchange rate**	Tells us how much of one currency you can exchange for another currency e.g. £1 = $1.39

KPI 9.12 Probability 1

1) Probability	How likely something is going to happen. All probabilities must be given as a fraction, decimal or a percentage (NOT a ratio).	**2) Probability scale words**	Impossible, Unlikely, Even chance, Likely, Certain
3) Probability scale	All probabilities exist between 0 and 1. Impossible = 0, Even chance = ½ and certain = 1 	**4) Systematic listing**	The outcomes for an event can be listed in an organised or systematic way to make sure that none of the possible outcomes are missed out.
5) Single event probability	The sum of the probabilities of a set of outcomes must equal one.	**6) Probability notation**	In probability we use the notation P(___) to represent the probability of something happening.
7) Probability of an event happening	$P(of\ an\ outcome\ happening) = \frac{number\ of\ the\ desired\ outcome}{total\ number\ of\ outcomes\ of\ the\ event}$	**8) Mutually exclusive**	Are events that cannot happen at the same time.
9) Expected outcomes	You need to multiply the probability by the total number of trials.	**10) Frequency trees**	Is used to record and organise information given as frequencies. This can then be used to calculate probabilities.
11) Sample space	Listing all of the possible outcomes from two events in a table. E.g. Displaying all of the scores for the sum of two spinners. Spinner A (1,2,3,4) and Spinner B (2,3,4)	**12) Relative frequency**	Relative frequency is used when probability is being estimated using the outcomes of an experiment or trial, when theoretical probability cannot be used. Relative frequency or experimental probability is calculated from the number of times an event happens, divided by the total number of trials in an actual experiment. $Relative\ Frequency = \frac{No.\ of\ Successful\ Outcomes}{No.\ of\ Trials}$

+		Spinner A			
		1	2	3	4
Spinner B	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

KPI 9.12 Probability 1

13) Venn Diagrams	A Venn diagram shows the relationship between groups of different outcomes.	**14) Element**	A list of numbers, objects or outcomes.
15) Universal set	Contains all of the elements for our question	**16) Set Notation**	**A** – all elements in A **A'** – all elements NOT in A **B** – all elements in B **B'** – all elements NOT in B
17) Intersection	**A∩B** – all the elements in both A and B 	**18) Union**	**A∪B** – all the elements in A or B or both 
19) Finding probabilities from a Venn diagram	Venn diagrams can be useful for organising information about frequencies and probabilities. We can then use them to solve conditional probability problems. E.g. The following Venn diagram shows information about the number of members of a local sport club who take part in the Archery and Badminton classes. A person is chosen at random. Find P(B only) $= \frac{18 + 23}{17 + 18 + 23 + 32} = \frac{41}{90}$		

KPI 9.13 Solving Equations 2

1) Solve	Use inverse operations to find the solution of an equation.	**2) Linear equation**	Contains an equals sign (=) and has one unknown. E.g. $5x - 2 = 2x + 7$
3) Equation	An equation is a statement with an equal sign, starting that two expressions are equal in value.		

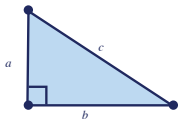
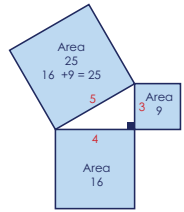
KPI 9.14 Inequalities 1

1) Representing an inequality on a number line– closed circle	A closed circle is used to show greater than or equal to (or less than or equal to) the number. $x \geq 3$ 	**2) Representing an inequality on a number line – open circle**	An open circle is used to show greater than (or less than) the number. $x > 3$ 
3) Reversing the Inequality	Multiplying or dividing both sides by a negative number reverses the inequality	E.g. $-3x < 6$ $x > -2$	

KPI 9.15 Sequences

1) Sequence	A pattern of numbers which fit a certain rule.	**2) Term**	A number in a sequence.
3) Term to term rule	The rule for how to get from one number to the next number in the sequence.	**4) Position**	Where a term is in a sequence.
5) Position to term rule	The rule for how to work out a number in a sequence if you know its position.	**6) Nth term**	Used to find a term in a sequence given its position e.g. 5n + 3
7) Linear sequence	The terms increase or decrease by the same amount each time. Also known as an arithmetic sequence. Nth term is written in the form, an + b.	**9) Geometric sequence**	A geometric sequence goes from one term to the next by always multiplying or dividing by the same value.
10) Fibonacci sequence	The Fibonacci sequence is unique because the next term is found by adding up the two previous terms. 1, 1, 2, 3, 5, 8, 13, 21…		

KPI 9.16 Pythagoras

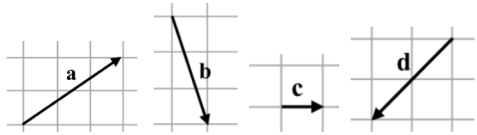
1) Right-angled triangle	A triangle that contains a right-angle (90 degrees).	**2) Hypotenuse**	The longest side – opposite the right-angle.
3) Pythagoras' Theorem	For any right-angled triangle, the area of the square of the longer length (the hypotenuse) is equal to the area of the squares of the shorter lengths added together.  $c^2 = a^2 + b^2$ $a^2 = c^2 - b^2$ $b^2 = c^2 - a^2$		

KPI 9.17 Interior and Exterior Angles			
1) Polygon	A polygon is a two-dimensional shape with 3 or more straight sides. A polygon is either regular or irregular: Regular – side lengths are equal, and all angles are equal. Irregular – side lengths are unequal, and angles are unequal.		
2) 3 sides	Triangle	**3) 4 sides**	Quadrilateral
4) 5 sides	Pentagon	**5) 6 sides**	Hexagon
6) 7 sides	Heptagon	**7) 8 sides**	Octagon
8) 9 sides	Nonagon	**9) 10 sides**	Decagon
10) 11 sides	Hendecagon	**11) 12 sides**	Dodecagon
12) Exterior Angles	Exterior angles of polygons sum to 360°. An exterior angle of a regular polygon is found by calculating $\frac{360}{n}$ n is the number of sides.	**13) Interior Angles**	In a regular polygon. Interior Angle + Exterior Angle = 180°
14) Tessellation	A pattern created with identical shapes that fit together with no gaps.		

KPI 9.18 Vectors 1

1) Vector

Vectors represent movement of a certain size in a certain direction, they are represented on a diagram with an arrow.



2) Magnitude

Magnitude is defined as the length of a vector.

3) Scalar

A scalar is the number we multiply a vector by.

4) Column vector

$\begin{pmatrix} a \\ b \end{pmatrix}$

a: movement along the x-axis (left/right)
b: movement along the y-axis (up/down)

5) Adding and subtracting column vectors

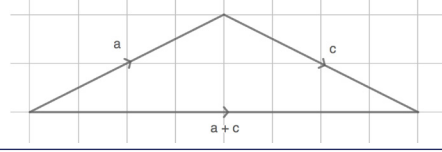
$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$$

6) Multiplying vectors

To multiply a column vector by a number, we multiply both values in the vector by that number.

7) Resultant vectors

The resultant vector is the vector that results from adding two or more vectors together.

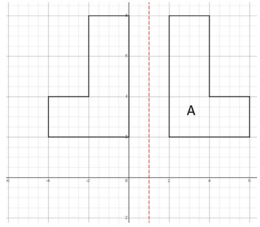
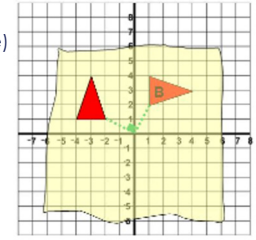
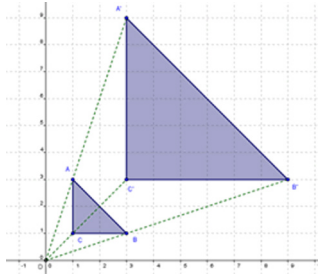
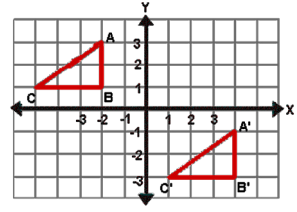


8) Parallel vectors

Travel in the same or opposite direction.
Can be of varying lengths.
Must be scalar multiples of one another.

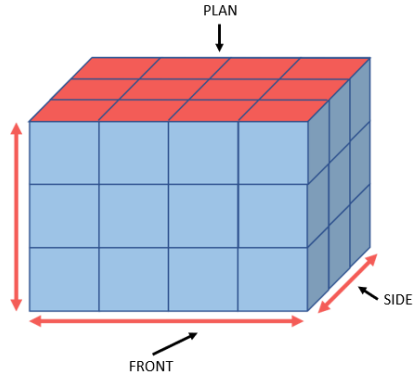
The vectors $\begin{pmatrix} 8 \\ 12 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ are parallel because $\begin{pmatrix} 8 \\ 12 \end{pmatrix} = 4\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

KPI 9.19 Transformations 1

1) Transformations	There are four types of transformations: Reflection Rotation Enlargement Translation	**2) Object**	This is the original shape used to perform the transformation on
3) Image	This is the new shape created as the result of the transformation.	**4) Congruent**	Two (or more) shapes that are the same size and the same shape.
5) Reflection	A shape is reflected in a line of symmetry. When a shape is reflected the image is always congruent to the object. The line of symmetry used must be given or found using an equation. An object and its image are always the same perpendicular distance from the line of symmetry. E.g. Reflect shape A in the line x=1 	**6) Rotation**	When rotating a shape we must have 3 pieces of information. Centre of rotation given as a coordinate (x,y) Angle of rotation (usually 90°,180°,270°) Direction (clockwise or anticlockwise) 
7) Enlargement	When enlarging a shape we must have two pieces of information. Centre of enlargement given as a coordinate (x,y) Scale factor 	**8) Translation**	A translation is a movement of an object When a shape in translated the image is congruent to the object. Translations are described using column vectors (a ¦ b) a: movement along the x-axis (left or right) b: movement along the y-axis (up or down) E.g. Translate the original triangle ABC by the vector (6 ¦ (– 4)) 

KPI 9.20 Plans and Elevations

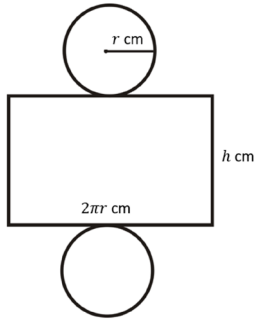
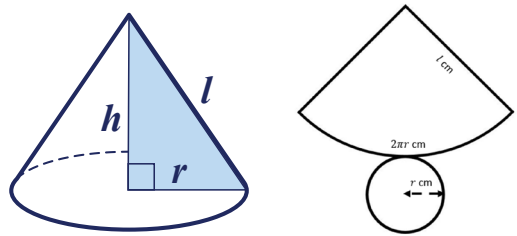
1) Plan	View looking vertically downwards.
2) Side elevation	View looking horizontally from the side.
3) Front elevation	View looking horizontally from the front.



KPI 9.21 Arcs and Sectors

1) Circumference	The perimeter of the circle. $C = \pi d$	**5) Area of a circle**	$A = \pi r^2$
2) Perimeter of a semi-circle	$P = \frac{\pi d}{2} + d$	**6) Area of a semi-circle**	$A = \frac{\pi r^2}{2}$
3) Perimeter of a quarter–circle	$P = \frac{\pi d}{4} + 2r$	**7) Area of a quarter-circle**	$A = \frac{\pi r^2}{4}$
4) Perimeter of a three-quarter circle	$P = \frac{3}{4}\pi d + 2r$	**8) Area of a three-quarter circle**	$A = \frac{3\pi r^2}{4}$

KPI 9.22 Surface Area

1) Surface Area	The total area of the surface of a three-dimensional object. For example, the surface area of a cube is the area of all 6 faces added together. It is measured in square units. E.g. square centimetres (cm^2), square metres (m^2).		
2) Cylinder	Surface Area = $2\pi r^2 + 2\pi rh$ 	**3) Cone**	Surface Area = $\pi r^2 + \pi rl$ 
4) Sphere	Surface Area = $4\pi r^2$	**5) Hemi-sphere**	Surface Area of a Hemi-sphere = $3\pi r^2$